CRESMET

# An Overview of a Project to Improve Mathematics and Science Education for a Technical Society: Cognitive Research Informs Curriculum Development and Instructional Support

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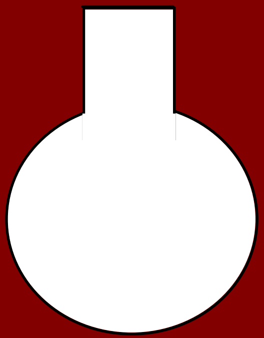
# The Bottle Problem

Imagine this bottle filling with water. Sketch a graph of the height as a function of the amount of water that's in the bottle



# Project Pathways

- Partnership of ASU, four school districts & Intel

  (ASU: Mathematicians, scientists, engineers, math and science educators, professional development experts)

Primary Goal:

- To produce a research-developed, refined & tested model of inservice professional development for secondary mathematics and science teachers.

Core Strategies:

- Four integrated math/science graduate courses + linked teacher professional learning communities (lesson study approach)

- Fifth course that introduces nano applications and research and societal implications

# Pathways Objectives for Teachers

- Deepen teachers' understanding of foundational mathematics & science concepts and their connections (function, rate-of-change, covariation, force, pressure)
- Improve teachers' reasoning abilities and STEM habits of mind (as defined in Carlson & Bloom problem solving framework and body of research on STEM "habits of mind")
- Support teachers in adopting "expert" beliefs about STEM learning, STEM teaching, and STEM methods (problem solving, scientific inquiry, engineering design)
- Improve ability to monitor, reflect on, & modify classroom instruction (K-12 and ASU)

# Overview

Pathways curriculum and instruction is based on research on learning.

- Development of cognitive frameworks that have informed the emergence of a Function Inventory (Thompson, 1994; Carlson, Jacobs, Coe, Larsen, Hsu, 2000; Oehrtman, Carlson & Thompson, in press)
- Development of a Problem Solving Framework (Carlson, 1998; Carlson & Bloom, 2005) that has informed the emergence of a STEM 'habits of mind' framework
  - A characterization of effective problem solving behaviors

# Instruments for Assessing Pathways Progress and Effectiveness

- Function Concept Inventory
  - Developed--instrument validation continues
- Beliefs about STEM habits and STEM teaching
  - Developed--early in the validation process
- PLC Observation Protocol
  - Still collecting qualitative data--not yet developed
- RTOP
  - Already validated and published

# Overview continued

Frameworks are developed from qualitative research that:

- Serve as a road map for course design, determiniation of hypothetical learning trajectory of student learning, curriculum design, classroom practices
- Guide the development of assessment tools and serve as a lens for analysis of data

# The Reflexive Relationship Between Individual Cognition and Classroom Practices

## One Example

**Determine/Revise Practices**

**Carlson & Bloom Problem solving framework**

-Describes effective mathematical practices

**Conceptual Frameworks (e.g., Covariation, FTC, Function)**

-Characterizes understanding (e.g., reasoning abilities, connections, notational issues)

**Theoretical grounding for**

-Designing curricular modules

-Determining course structure

-Determining classroom norm

**Lens for researching emerging practices and understandings**

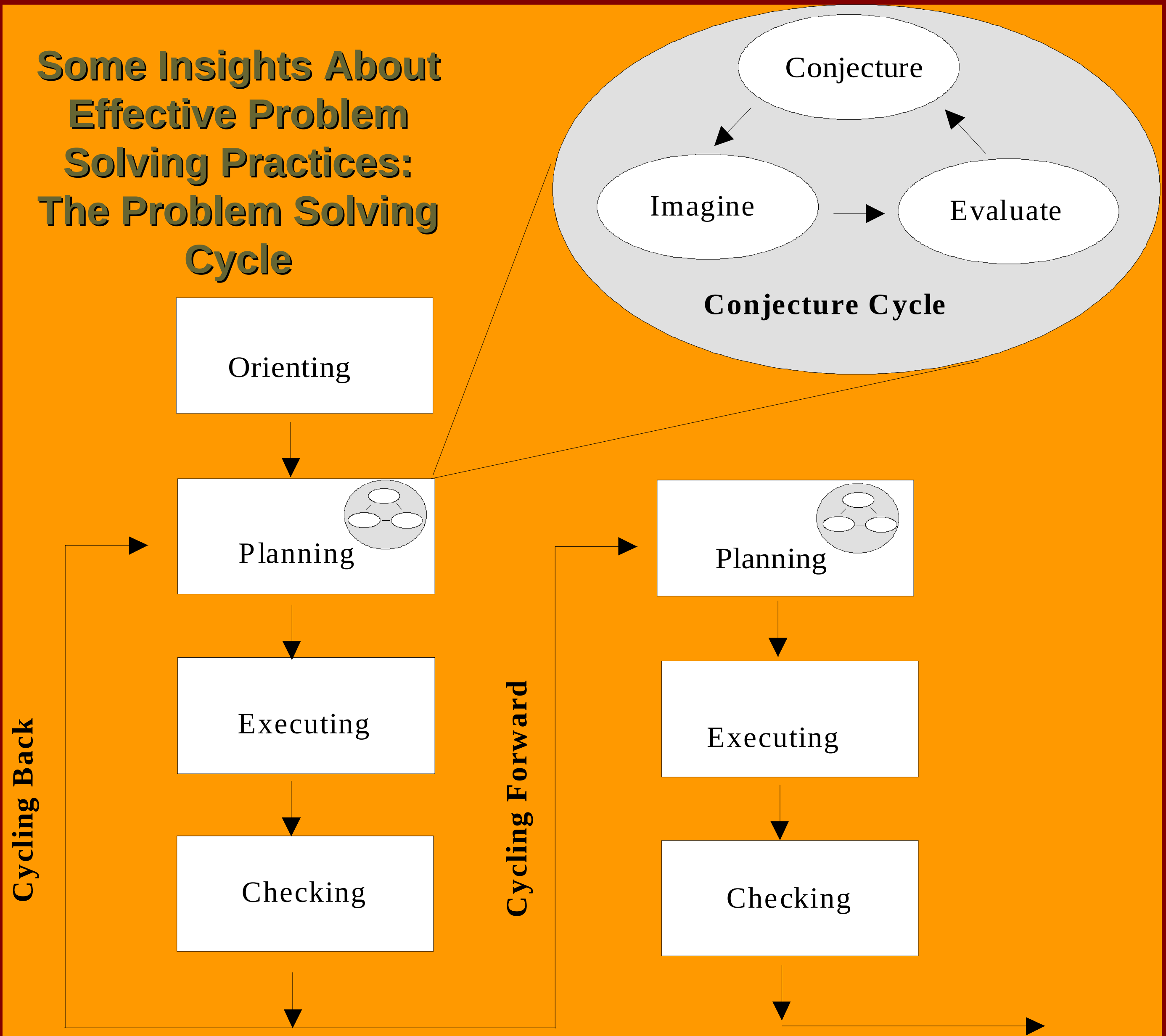
**Revise Cognitive Frameworks**

# Problem Solving Research Informs Individual Cognition and Classroom Practices

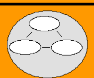
- **Investigation of the mathematical behaviors and practices of mathematicians revealed new insights about the mathematical practices of effective problem solvers**
  - Analysis of consistent patterns led to the development of the Multidimensional Problem Solving Framework (Carlson and Bloom, in press; Educational Studies in Mathematics)

- **This MPS framework provides useful characterization of individual mathematical practices that should be supported and promoted**
  - Provides the end point for a hypothetical learning trajectory relative to effective problem solving behaviors/mathematical practices
  - Useful for designing classroom and curricular experiences, both individual and community

# Some Insights About Effective Problem Solving Practices: The Problem Solving Cycle



# The Multidimensional Problem Solving Framework: A Characterization of Effective Problem Solving Practices

| Phase Behavior | Resources | Heuristics | Affect | Monitoring |
|---|---|---|---|---|
| **Orienting<br>Sense making<br>Organizing<br>Constructing** | **Mathematical concepts, facts and algorithms were accessed when attempting to make sense of the problem; prob. categorization** | The solver often drew pictures, labeled unknowns, and classified the problem. (Solvers were sometimes observed saying, “this is an X kind of problem.”) | Motivation to make sense of the problem was influenced by their strong curiosity and high interest. High confidence was consistently exhibited, as was strong mathematical integrity. | **Self-talk and reflective behaviors helped to keep their minds engaged. The solvers were observed asking “What does this mean?”** |
| <br>**Planning<br><br>Conjecturing<br>Imagining<br>Evaluating** | **Conceptual knowledge and facts were accessed to construct conjectures and make informed decisions about strategies and approaches.** | Specific computational heuristics and geometric relationships were accessed and considered when determining a solution approach. | **High level of mathematical integrity was displayed; Never offered a solution that did not have a logical foundation; never pretended to know when he didn’t** | Solvers reflected on the effectiveness of their strategies and plans. They frequently asked themselves questions such as, “Will this take me where I want to go?” and “How efficient will Approach X be?” |
| **Executing<br>Computing<br>Constructing** | Conceptual knowledge, facts, and algorithms were accessed when executing, computing, and constructing. Without conceptual knowledge, monitoring of constructions was misguided. | Fluency with a wide repertoire of heuristics, algorithms, and computational approaches were needed for the efficient execution of a solution. | **Intimacy with the problem, integrity in constructions, frustration, joy, defense mechanisms, and concern for aesthetic solutions emerged in the context of constructing and computing.** | Conceptual understandings and numerical intuitions were employed to reflect on the reasonableness of the solution progress and products when constructing solution statements. |
| **Checking<br>Verifying<br>Decision making** | Resources, including well-connected conceptual knowledge, informed the solver as to the reasonableness or correctness of the solution attained. | Computational and algorithmic shortcuts were used to verify the correctness of the answers and to ascertain the reasonableness of the computations. | As with the other phases, many affective behaviors were displayed. It is at this phase that frustration sometimes overwhelmed the solver. | Reflections on the efficiency, correctness, and aesthetic quality of the solution provided useful feedback to the solver. |

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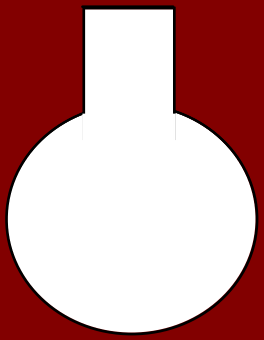


# What Reasoning Abilities are Needed to Graph Height as a Function of Volume?

***Covariational reasoning*:**

**Imagining two quantities changing in tandem while attending to how they change in relation to each other (Thompson, 1994; Carlson et al., 2002)**

**-Is needed for interpreting and representing models of dynamically changing events**

**-Is foundational for understanding major concepts of calculus (limit, derivative, accumulation, the FTC)**

**-Is necessary for forming dynamic images of real world phenomena and for meaningful modeling of this phenomena with formula, graphs, table.**

# *Mental Actions of the Covariational Reasoning Framework*

***MA1)*** ***Coordinating one variable with changes in the other variable.***

***MA2)*** ***Coordinating the direction of change in one variable with changes in the other variable*** **(e.g., increasing, decreasing);**

***MA3)*** ***Coordinating the amount of change of one variable with changes in the other variable.***

***MA4)*** ***Coodinating the average rate of change of one variable (with respect to the other variable) with uniform changes in the other variable.***

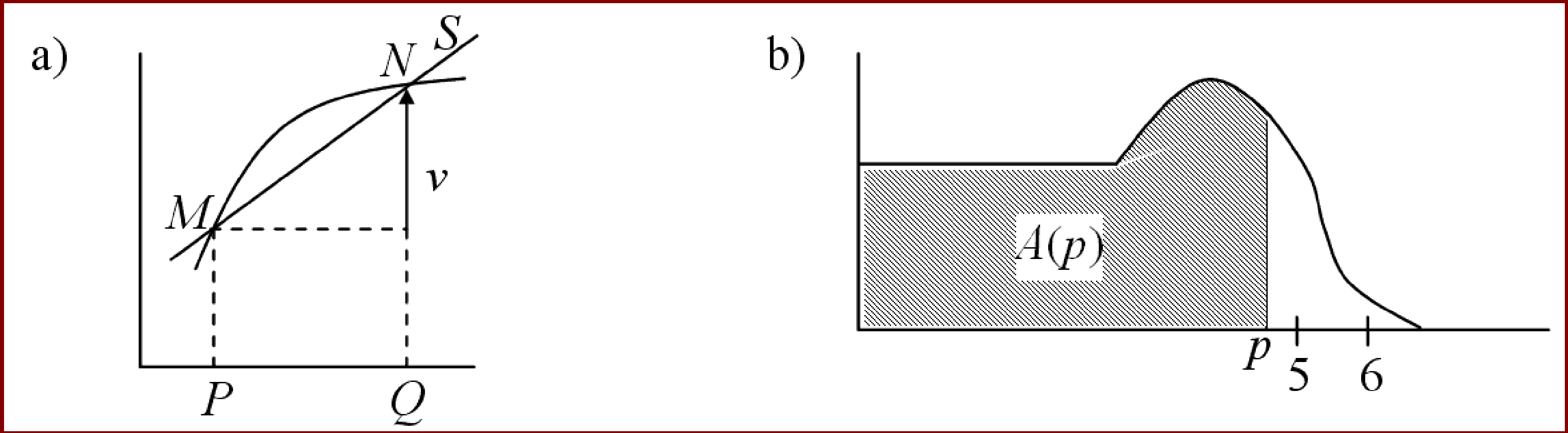
***MA5)*** ***Coordinating the instantaneous rate of change of one variable (with respect to the other variable) with continuous changes in the other variable.***

***(Carlson, Jacobs, Coe, Larsen, Hsu, 2002)***

# Covariational Reasoning:
# A Foundational Reasoning Ability for:

## Derivative

## Accumulation




$$A(p) = \int_0^p f(t)\,dt$$

# Cognitive Frameworks Inform Teaching and Support Research in Instructional Settings

- Planning of course materials: Lesson logic maps out the unfolding of the content/concepts, worksheets based on cognitive frameworks
- Classroom interaction patterns: students are expected to offer up logical conjectures; students are expected to make sense of others reasoning and not pretend to understand if they don't
- The nature of the homework and class projects

# Closing Remarks

- Use of individual cognitive models of learning or knowing, such as the *Covariation Framework* and the *Multidimensional Problem Solving Framework* increase the purposefulness of instructional materials and instructional actions

- Cognitive frameworks are necessary for investigating the effectiveness of the implementation
- Mathematical concepts, reasoning abilities and problem solving behaviors provide greater power for solving engineering related problems

Call for research on learning and using nano concepts to build on methods and body of knowledge in math and science education research.